Modeling of pneumatic multi-chamber bending actuators benefits from a well-chosen design - and vice versa*

Frederik Lamping and Kristin M. de Payrebrune[1]

Abstract— **Designing as well as modeling is challenging in soft robotics. In this study, we propose thinking both topics together. This can be done by already taking the modeling into account when designing. Vice versa, the model can be used to tune the design parameters to fit a desired task. As demonstrated using the example of a pneumatic multi-chamber actuator, an efficient and an easy-to-model design both fulfill equal demands. The analytic model of the actuator is simple and allows for interpretation of the influence of design parameters. This is a promising result which encourages applying it to soft robot designs using other motion mechanisms.**

I. Introduction

Designing and modeling soft actuators is an important field in soft robotics. The variety of designs is large and continuous to grow. As many examples from literature show, novel designs are often presented accompanied by an experimental investigation, e.g. in [1], or by a model, e.g. in [2] and [3]. Conversely, new modeling methods are often related to an existing design, e.g. in [4] and [5]. However, in both cases, either the focus is on the design or on the model.

Our approach is to take into account the interdependence of designing and modeling, i.e., to choose a design that is beneficial for modeling, and to choose a model that provides important information on design parameters. We demonstrate this idea using the example of a pneumatic multi-chamber actuator. The design is subject of section II, while the model is subject of section III. Finally, we conclude our findings in section IV.

II. Design

In a recent study of the authors [6], a design for a modular pneumatic multi-chamber bending actuator is derived by systematic investigation. As shown in Fig. 1, the actuator consists of three parallel cylindrical chambers bonded by end caps and a linking element between the end caps. Multiple modules can be stacked by replacing one of the end caps with a connector cap. The chambers are reinforced by rings in order to reduce the manufacturing effort compared to a fiber reinforcement. However, since the distance between rings is relatively large, ballooning occurs at higher pressure.

The study comprises three design aspects that have an important impact on modeling: the shape of the cross section of the chambers, their reinforcement and their bonding.

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[1] The authors are with the Institute for Computational Physics in Engineering, Technische Universität Kaiserslautern, Germany
frederik.lamping@mv.uni-kl.de
kristin.payrebrune@mv.uni-kl.de

A. Cross section

Three examples for different shapes of cross section are shown in Fig. 2. A comparison of the bending angle that actuators with these cross sections achieve, shows that a turned semicircular and a circular cross section perform approximately equal [6]. However, the semicircular cross section deforms to circular when pressurized, which leads to high stresses at the former edges [7]. Therefore, the cross section of multi-chamber bending actuators should be chosen circular.

This has a positive side effect on modeling. Independent from the model, a simple geometry, e.g. circular, is advantageous since it simplifies finding the kinematics of the chambers.

B. Reinforcement

The reinforcement prevents the chambers from ballooning when pressurized. Rather, the chambers are forced to elongate.

In a preliminary study of the authors [8], the stretch behavior of individual cylindrical chambers is investigated for different reinforcements by using an analytic model. A typical technique for reinforcing pneumatic actuators is wrapping fibers around it. The analytic model indicates that the fiber angle has a significant influence on the linearity between pressure and stretch. Another technique is using rings as done by the actuator in Fig. 1. Here, the distance between rings has a significant influence on the linearity between pressurization and stretch, see Fig. 3.

C. Bonding of chambers

The working principle of multi-chamber actuators is transforming the elongation of the individual chambers into a bending of the whole actuator by a parallel alignment. In general, this can be achieved by two techniques: either embedding the chambers into a body of silicone, or using linking elements in order to bond individually manufactured chambers. The latter provides better bending performance since any dispensable material opposes bending. As long as the distance between linking elements is small enough to prevent buckling, they can be both stiff or flexible.

Similar to a circular cross section, using linking elements has also a positive side effect on modeling since the geometric complexity is reduced. The linking elements can be considered rigid or semi-rigid, respectively, in the model, i.e. rigid elements only have an influence on the kinematics but do not deform.

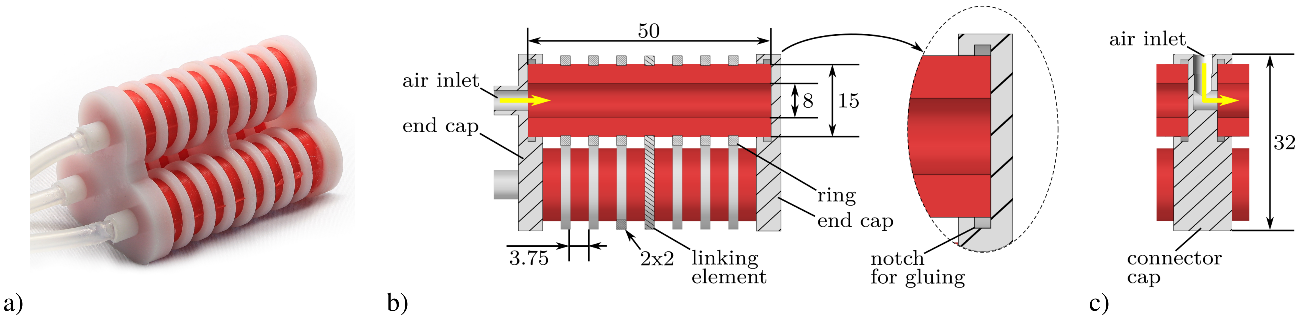

Fig. 1: Modular actuator assembled a), its dimensions and name of its components b), and a connector cap that can be used instead of an end cap to stack modules c). The sideview shows a cut through the middle plane of the actuator.

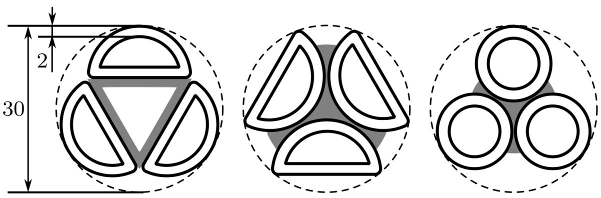

Fig. 2: Actuators with semicircular, turned semicircular, and circular shaped cross sections (rigid linking elements between chambers are indicated gray) [6]

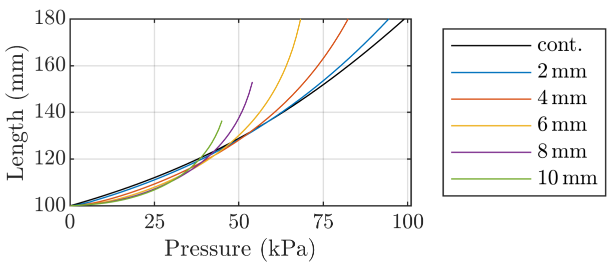

Fig. 3: Stretch of a pneumatic cylinder as a function of pressure for continuous reinforcement and varying distance between rings [6]

III. Model

A common technique for modeling slender soft actuators is the Cosserat beam theory which is used by many, e.g in [4]. Due to a reduction of the three-dimensional actuator to a one-dimensional beam reduces computational costs. The Cosserat beam theory is of special interest in soft robotics since it covers not only bending, but also elongation, torsion and shear.

In the first part of this section, we introduce the Cosserat beam theory and its most important parameters. In the second and third part, we introduce a novel method for the analytic derivation of these parameters for multi-chamber bending actuators, and offer an interpretation of the equations derived.

A. Cosserat beam theory

The Cosserat beam theory defines the beam as a curve $\mathbf{r}(s)$ in space, where s is the arc-length parameter, see Fig. 4. The orientation of each point of the curve can be expressed

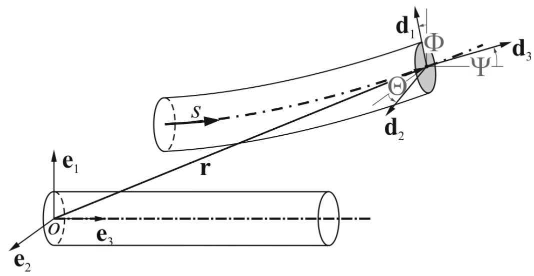

Fig. 4: Coordinate systems of the Cosserat beam

either by a global coordinate system (e_1, e_2, e_3) or by local directors (d_1, d_2, d_3), where (Φ, Θ, Ψ) express the rotation between both systems.

For the statical case, the classical form of equations for the special theory of Cosserat beams is

$$\mathbf{n}' + \mathbf{f} = \mathbf{0} \tag{1}$$
$$\mathbf{m}' + \mathbf{v} \times \mathbf{n} + \mathbf{l} = \mathbf{0}, \tag{2}$$

where $\mathbf{v}$ is the tangent of $\mathbf{r}$, $\mathbf{f}$ and $\mathbf{l}$ are the body force and body moment per length, $\mathbf{m}$ comprises the bending and twisting moments, and $\mathbf{n}$ comprises the shear forces and the axial force.

In this study we assume that the body forces and moments are defined by the constitutive relations

$$\mathbf{n} = \mathbf{K}(\mathbf{v} - \mathbf{d}_3) \tag{3}$$
$$\mathbf{m} = \mathbf{J}(\mathbf{u} - \mathbf{u}_0), \tag{4}$$

where $\mathbf{u}$ measures flexure and twist, respectively. According to the classical linear approaches in beam theory we define $\mathbf{K}$ as

$$\mathbf{K} = K_{ij}(\mathbf{d}_i \otimes \mathbf{d}_j) \tag{5}$$

$$K_{ij} = \begin{bmatrix} kGA & 0 & 0 \\ 0 & kGA & 0 \\ 0 & 0 & EA \end{bmatrix},$$

with kGA the shear stiffness and EA the extensional stiff-

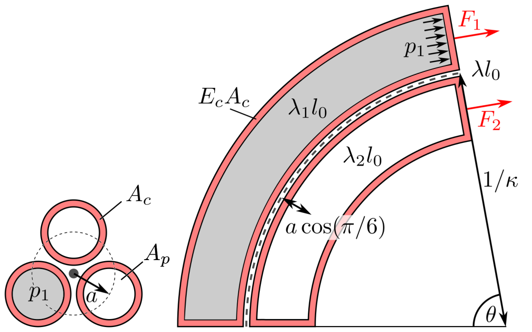

Fig. 5: Scheme of a multi-chamber bending actuator

ness. Analogous, we define $\mathbf{J}$ as

$$\mathbf{J} = J_{ij}(\mathbf{d}_i \otimes \mathbf{d}_j) \tag{6}$$

$$J_{ij} = \begin{bmatrix} EI & 0 & 0 \\ 0 & EI & 0 \\ 0 & 0 & GI \end{bmatrix}$$

with EI the bending stiffness and GI the torsional stiffness.

In summary, the following parameters need to be known for a full description of the beam:

- extensional stiffness EA
- bending stiffness EI (two directions)
- shear stiffness κGA (two directions)
- torsional stiffness GI

The parameter derivation in the next part focuses on the extensional stiffness EA and the bending stiffness EI.

B. Parameter derivation

For deriving the parameters of the beam, we assume an actuator with three identical cylindrical chambers bonded by linking elements as shown in Fig. 5. The distance between the chambers always remains equal. The stretch ratio of each individual chamber λ_i is

$$\lambda_i = (E_c A_c)^{-1} \cdot (p_i A_p + F_i) + 1, \tag{7}$$

where $E_c A_c$ is the extensional stiffness of the chambers and F_i is the axial force. The pressurization of the individual chamber p_i with the area of attack A_p also induces an axial force.

We assume that the bending stiffness of an individual cylinder is negligible compared to its extensional stiffness. This can be shown by regarding the energy of the system, but is beyond the scope of this study. Additionally, we assume that the actuator has a constant curvature. This assumption is not necessary anymore once the parameters of the beam are found. With these assumptions and the distance between chambers a, we find a kinematic relation between the stretch λ and the curvature κ of the bending actuator, and the length of the individual chambers

$$\lambda_i = \lambda_i(\lambda, \kappa) \tag{8}$$

We can find the length and the curvature of the bending actuator by substituting Equation 8 in Equation 7. By definition, the extensional stiffness is the relation between axial force and axial stretch, and the bending stiffness is the relation between bending moment and curvature. Relating an arbitrary force and moment to the resulting deformation vanishes the former and we find that the extensional stiffness EA and the bending stiffness EI of the actuator are

$$EA = 3 \cdot E_c A_c \tag{9}$$

$$EI = 1.5 \cdot E_c A_c \cdot \lambda \cdot a^2. \tag{10}$$

Please note that these parameters are universal without being restricted to constant curvature. Furthermore, the method can be extended to geometrically more complex actuators, e.g. with chambers embedded in silicone, but with higher errors due to stronger simplification of the geometry, and to actuators with a different number of chambers. In the derivation presented, the extensional stiffness is assumed as linear, but nonlinear extensional stiffness would be possible as well.

C. Interpretation

Due to the simple formulation of Equation 9 and Equation 10, interpretation of the equations becomes possible. The results of the interpretation can be used to tune the design in section II such that it efficiently fulfills a desired task.

1) Extensional stiffness: According to Equation 9, the extensional stiffness of the bending actuator EA is a multiple of the extensional stiffness of its individual chambers $E_c A_c$. The factor is similar to the number of chambers n.

As already mentioned, the extensional stiffness in this study is assumed to be linear, which needs not necessarily to be the case. There is no requirement for the method of determining the extensional stiffness. It can be done analytically as in a preliminary study of the authors [8] and many others, or by experimental investigation. For all methods, findings on individual chambers are directly applicable to multi-chamber bending actuators.

2) Bending stiffness: Similar to the extensional stiffness, the equation of the bending stiffness contains a factor, which is 1.5 in Equation 10. This factor also depends on the number of chambers n and is $n/2$ if $n > 2$.

Since the bending stiffness of the individual chambers is neglected in the derivation, the bending stiffness of the whole actuator depends only on the extensional stiffness of the individual chambers. This is in accordance with the visible behavior of such actuators that bending always coincides with stretching. Thus, findings on the extensional stiffness of individual chambers are not only applicable to the extensional stiffness of multi-chamber actuators, but also to their bending stiffness.

The dependency of the bending stiffness on the current length of the actuator λ is an important finding. Bending is caused by a difference of length of the chambers. The larger the actuator stretches, the larger the difference in length must be to achieve the same curvature. Consequently, the bending moment must increase as well.

Obviously, the spatial arrangement of the chambers has an influence on the bending stiffness. This is taken into account by the distance between the central points of the chambers and of the actuator a.

Interestingly, the bending stiffness is independent from the direction of bending as long as the number of chambers is three or higher. This can be explained by the spatial arrangement of the chambers in a circle.

IV. Conclusion

In this study, we propose taking the interdependence of designing and modeling into account in soft robotics. A pneumatic multiple-chamber actuator serves as an example for the idea.

In section II, we introduce a multi-chamber bending actuator that is subject of a recent study of the authors. Hereby, we highlight those design aspects with a significant influence on modeling. A simple beam model of the actuator, which benefits from the design, is introduced in section III. The extensional stiffness and the bending stiffness of the bending actuator can be derived from the extensional stiffness of its individual chambers. Since the equations derived are simple, interpretation of the individual parameters having an influence on the extensional stiffness and bending stiffness becomes possible. The designing of the bending actuator benefits from this interpretation, since tuning its parameters to efficiently fulfill a desired task becomes possible.

Future work comprises two aspects. On the one hand, finding similar methods in order to derive the shear stiffness and torsional stiffness. Both parameters are necessary for a full description of the beam. On the other hand, making the method as universal as possible by adding non-linear materials and more complex geometries.

References

[1] M. Cianchetti, T. Ranzani, G. Gerboni, I. De Falco, C. Laschi, and A. Menciassi, "Stiff-flop surgical manipulator: Mechanical design and experimental characterization of the single module," in *2013 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, 2013, pp. 3576–3581.

[2] J. Bishop-Moser and S. Kota, "Design and modeling of generalized fiber-reinforced pneumatic soft actuators," *IEEE Transactions on Robotics*, vol. 31, no. 3, pp. 536–545, 2015.

[3] Y. Toshimitsu, K. W. Wong, T. Buchner, T. Buchner, R. K. Katzschmann, and R. K. Katzschmann, "Sopra: Fabrication & dynamical modeling of a scalable soft continuum robotic arm with integrated proprioceptive sensing," 2021.

[4] D. Trivedi, A. Lotfi, and C. D. Rahn, "Geometrically exact dynamic models for soft robotic manipulators," in *2007 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, 2007, pp. 1497–1502.

[5] S. M. Mustaza, Y. Elsayed, C. Lekakou, C. Saaj, and J. Fras, "Dynamic modeling of fiber-reinforced soft manipulator: A visco-hyperelastic material-based continuum mechanics approach," *Soft Robotics*, vol. 6, no. 3, pp. 305–317, jun 2019.

[6] F. Lamping and K. M. de Payrebrune, "A systematically derived design for a modular pneumatic soft bending actuator," in *2022 5th IEEE International Conference on Soft Robotics (RoboSoft)*. IEEE, apr 2022, in press.

[7] J. Fraś, "Depth-map-based shape recognition of soft continuum manipulator body," in *Automation 2017*. Springer International Publishing, 2017, pp. 300–307.

[8] F. Lamping and K. M. de Payrebrune, "A virtual work model for the design and parameter identification of cylindrical pressure-driven soft actuators," *Journal of Mechanisms and Robotics*, vol. 14, no. 3, nov 2021.